Examples of Fire Safety Engineering calculations.

1 A note on my calculations

The calculations presented here are intended to give the reader a small impression of the kind of problems that are amenable to calculation in the field of fire safety engineering. Some are simple, some complex. The calculations are related to fire safety in buildings and to the safe relation of buildings to other risks.

The calculations are hand-calculations and all can be made using a hand-held scientific calculator or, better still, a spreadsheet such as Excel. The theoretical or empirical background to the equations used is not discussed but it is hoped that having seen the potential for calculation that the reader will be encouraged to read the scientific literature and find the limits of application of the equations used. Some limits of application are given in the various parts of BS 7974 ‘Application of fire safety engineering to the design of buildings’.

Of course, many problems are not amenable to hand calculation and this applies especially to complex problems of fire effluent movement where simplified zone models are sometimes wrongly used, and also in the field of mass-people movement. In both cases the problem may be better solved by use of powerful computer-aided techniques based, in the case of complex fire effluent movement, on computational fluid dynamics (CFD). The results of such techniques are difficult to check but the use of a hand-calculation can sometimes be used to see if the answer given by the complex method ‘looks sensible’.

More information on FSE calculations are given in BS 7974 and in the work of ISO TC 92 SC4.

2 Smoke control calculations

The smoke control designer has two design options A and B. In option A it is assumed that steady state fire conditions are present (as in a sprinkler-controlled fire or an unsprinklered fully developed fire which has reached a steady state heat release rate). In this case the designer may decide to provide natural ventilation or mechanical smoke extract solely to maintain the smoke layer at an acceptable constant height for infinite time. This option may be adopted when it is agreed in the QDR that untenable conditions must not be allowed to occur at any time – as in a large public assembly space or in an industrial facility where the fire service attendance time is unavoidably very large. In option B it is assumed that the fire is continuously developing so that the rate of heat release is increasing with time (e.g. as in a t-squared design fire). Option A leads to a larger area of natural smoke vents or larger capacity fans for mechanical extraction and this is normally more costly than option B. Option A is a very
conservative option as it ignores the smoke produced in the smouldering and development phase and the associated time taken to get to full development. Most present day design is based on Option B.

A typical plot of thickness and temperature of smoke layer with time (or any other derivative eg optical density, toxicity etc), is shown below:

![Diagram](image.png)

### 2.1 Time to smoke fill a portal framed compartment assuming a $t^2$ growing fire and no roof ventilation

A 2-bay, low-pitch portal framed building has a valley gutter and ridges at 12m and 13m from floor level respectively. The building is 40m wide and 60m long. It has an exhibition item in the centre which, if ignited, is expected to burn in a way similar to a fast $t^2$ fire. See figure below. There are no vents in the roof, but there are doors which open on detection of smoke, and these are assumed to allow adequate supply of air at low level to feed the fire.

Assume two-thirds of the total heat released is convected heat in the plume and that there is no heat loss from the building enclosure.

Determine the rate at which the compartment will fill with smoke, and find the time for the smoke to fill down to within 5m and 2m of floor level. Determine if occupants are safe if they leave the building within 3 minutes of ignition.
Figure. Cross section through portal framed building

Data given:
- average height of enclosure = 12.5m i.e. \((12+13)/2\)
- length of building = 60m
- width of building = 40m
- ambient air temperature, \(T_0 = 15^\circ C\)
- density of air, \(\rho = 1.2kg/m^3\)
- specific heat of air \(C_p = 1.92\)
- clear air layer needed = 6m
- fire growth parameter for fast \(t^2\) fire \(a = 0.0444\) kW/s²
- height of virtual source above base of fire, \(z_o = 0\)

From the literature (e.g. CIBSE Technical Memorandum TM19: 1995):

\[
Q = 1000 \left( \frac{t}{t_g} \right)^2 \quad \text{(CIBSE Eqn 4.1)} \tag{1}
\]

where \(t_g\) = characteristic growth time. For a fast fire \(t_g = 150\) and Equation (1) becomes

\[
Q = 0.0444t^2 \quad \text{(2)}
\]

\[
M = 0.071Q_{p}^{1/3}(z - z_o)^{5/3} \quad \text{(CIBSE Eqn 5.6)} \tag{2}
\]

\[
T_m = \frac{Q_p}{MC_p} + T_o \quad \text{(CIBSE Eqn 5.17)} \tag{3}
\]

\[
v = \frac{MT_m}{\rho_oT_o} \quad \text{(CIBSE Eqn 5.19)} \tag{4}
\]

A spreadsheet is prepared which calculates smoke layer temperature and thickness as a function of time.
First we assume the worst condition, that the plume height $z$ remains constant as time progresses (in reality the plume height reduces as the thickness of smoke layer increases and thus the height over which air entrainment takes place reduces with time). Substituting data gives:

- time for smoke layer to reach down to 5m from floor level = 325sec
- time for smoke layer to reach down to 2m from floor level = 488sec

Assume now that the air entrainment reduces as the smoke layer increases in thickness i.e. $z$ reduces. We now have:

- time for smoke layer to reach down to 5m from floor level = 534sec
- time for smoke layer to reach down to 2m from floor level = 700sec

So we see the large increase in smoke fill time, i.e. from 488 to 700 seconds for smoke to reach to within 2m of floor level. Thus using constant $z$ is conservative – if occupants could evacuate within this period they would be safe.

We are asked if the occupants are safe to complete their evacuation by 3 minutes from ignition. From the calculation the smoke layer is approximately 2m deep (well above head height) and at a temperature of 34°C for constant $z$ and at 38°C for reducing $z$, at this time. The conditions are clearly tenable.

### 2.2 Time to smoke fill an enclosure with sloping ceiling etc

Often, in a multi-zone model used for calculating the rate of smoke fill in an enclosure, the enclosure will have a flat roof or ceiling and this is the simplest geometry to model because the plan shape taken by the lower surface of the smoke layer does not change as the smoke fills the enclosure from top downwards.

Figure 1A shows a simple rectangular enclosure with an axi-symmetric fire plume located at the centre of the floor. Figure 1B shows an enclosure of the same height but with a 45-degree sloping roof (some modern atria are of this shape). Assuming that each enclosure has the same width and the same length and fires with the same rate of heat release, the only variable is the shape of the roof. The enclosure with the inclined roof will fill more rapidly, and there will be less escape time for occupants, especially if they are located above the floor of the enclosure eg on a balcony. (Figures 1A and 1B have been shown with roughly the same amount of smoke in the layer). In the enclosure shown in Figure 1B the height of the vertical plume is much smaller and therefore the amount of air entrained is far less and the smoke entering the layer will be hotter than in Figure 1A.
At an early stage in the fire development in the enclosure with a sloping roof, Figure 1C, there may be the temporary formation of a sloping plume/jet. If the angle of slope is large there may be a small amount of air entrained into the sloping plume and it is a matter of engineering judgement if this can be ignored. Note that when the slope is zero the amount of air entrained in the layer spreading below the ceiling is assumed to be zero – this is an explicit assumption in the ceiling jet equation. If the roof is steeply inclined, the height over which air is entrained is different on the ‘opposite surfaces’ of the plume so that an average height should be adopted in the plume equation. Again, the greater the plume height the greater the entrainment.

In the non-flat roof condition the location of the fire needs to be considered. In Figure 1D it is important to consider this fact and assume the worst fire location (fire at position 2) if a sensitivity analysis is not made: if the fire is near the tall wall more air will be entrained in the vertical axi-symmetric plume and the layer depth will initially increase more rapidly than in the same fire located near the short wall. Note that if the plume contacts the wall before reaching the smoke layer the axi-symmetric plume equation can
only be used if the entrainment factor is reduced; it is reduced by 50% if the fire is located directly against the wall thus assuming that air is entrained from only one side of the plume through its full height.

When making the smoke fill calculation it is normally necessary to represent the roof shape as a series of rectangular shapes. Figure 1E represents part of a sloping roof at the top, in which each rectangle has the same area as the actual smoke filled shape at that level (ie w1.h1 represents the area of shaded triangle 1). In this way it is possible to integrate the volume of smoke produced incrementally. A curved roof would be represented as a number of rectangles, Figure 1F.

In the process of idealizing the sloping or curved volume as a series of rectangular volumes the engineer should keep in mind the physics of smoke and air entrainment and also the effects of heat losses from the enclosure surfaces. The following are some aspects which should be considered and their effect may need to be quantified in sensitivity analyses:

a) For a given heat release rate from the fire, the greater the heat loss from the enclosure (e.g. due to low thermal insulation of the walls and ceiling), the lower the temperature of the smoke layer, and the greater the smoke layer thickness.

b) For the sloping roof, the axi-symmetric plume height is reducing more rapidly and therefore less air is entrained and the smoke layer temperature is higher than in the case of the enclosure with a flat roof. The higher the smoke layer temperature, the more radiation produced by it.

c) the higher the plume the greater the amount of air entrained

The engineer must consider the effect of any partial barriers to the flow of smoke. In Figure 1G a balcony is present at B. The smoke fill time for a fire on the floor away from the balcony, Figure 1H, will be greater than if the fire is assumed to occur under the balcony, and this is because the spill plume formed by the balcony has a larger entrainment factor than the axi-symmetric plume (compare the levels in Figures 1G and 1H). Obstructions to horizontal smoke flow, e.g. the downstand beam located at C in Figure 1J, also needs to be considered as such obstructions will increase the entrainment and lead to a lower temperature in the smoke layer remote from the fire, perhaps leading to stratification and loss of buoyancy at large distances.

The following calculations on sloping roofs were made as part of work of ISO TC 92 SC4 and was originally presented in a PowerPoint presentation dated October 2009.

Building is 80m long and has cross section as shown below (40m high by 40m wide). It has a sloping roof. Design fire is steady state 5MW from time zero.

In one analysis the roof portion is idealised as 4 rectangles as shown here. In other analyses it is idealised as one rectangle of equivalent area.
Two conditions considered – central plume and offset plume – as below

Now, see below, the roof is idealised as a single rectangle (40m by 10m) of equivalent area
Offset plume is 10m from wall. Average plume height is 22.5m before smoke builds down to this level. Entrainment into sloping plume is ignored. Smoke fills rectangular spaces - early stage shown left, later stage shown right below

The axi-symmetric plume equation is used as below

\[ M = 0.071Q_p^{1/3}z^{5/3} \]

Model assumptions are as follows:

- The virtual source term, \( z_0 \), is assumed to be zero, which is often an assumption in practical design.
- Plume/wall interaction is ignored
- Heat losses to the enclosing surfaces are ignored.
- Effects such as stratification are also ignored.
- For the offset plume the entrainment in sloping plume early on is zero. This entrainment could be accounted for.
The calculations have been carried out using an Excel spreadsheet written for the purpose. This uses a volume integration method and has been made using time steps of 1 second. Smoke layer depth is plotted against time. Convective heat release rate is assumed to be 2/3 of the 5MW total heat release rate.

Calculation procedure
- Decide time step e.g. 1 second in these calculations
- For each time step do the following:
  - State total heat release rate, \( Q_t \) (kW)
  - Calculate convective heat release rate, \( Q_p \) (kW)
  - Calculate rate of smoke mass, \( M \) (kg/sec)
  - Calculate absolute temperature of smoke, \( T_m \) (K)
  - Calculate increment in smoke volume, \( \Delta V \), (m³)
  - Calculate total volume of smoke by integrating the smoke volume increments, \( V \) (m³)
  - Calculate smoke layer thickness, \( \delta \) (m) and, if needed, clear layer height

Manual intervention in the spreadsheet is necessary where:
1. the plume height changes from a constant to a variable (as in the offset plume calculations)
2. where the plan area of the smoke reservoir changes (e.g. when the smoke enters a new rectangle)
Results are plotted as shown above and the following conclusions can be drawn:

- There appears to be little effect on smoke layer depth of a) rectangle shape used to represent the sloping roof or b) position of plume.
- The lowermost curve is for smoke layer thickness for a roof which is 30m high. The other curves are for a 40m roof height (to the top of roof).
- Plotting clear air layer thickness (i.e. 40m minus smoke layer thickness) gives a more meaningful comparison as shown in next graph.

This following graphs show the clear layer height comparison. There is only a small difference in smoke fill rate for both the rectangle assumptions (compare shaded areas).

Effect of time step is shown below from which it appears to have little effect.

We now look at a triangular smoke fill space.
This assumes the roof in the previous examples is the building.
Smoking fill rates for central plume and offset plumes are presented in next figure.
Again the entrainment in the sloping plume is ignored.

Results are plotted below.

Conclusions (for building geometry and plume positions adopted) are:

1. It appears that the idealised shapes assumed for the roof have little effect on smoke fill time if the clear layer height is of principal interest. This applies, surprisingly, for central plume and offset plume conditions for the whole building. When the roof alone is the building the position of plume does affect fill, as one would expect from intuition.
2. In the integration of volume it appears that time step is not significant (for 1sec and 10sec).

### 3 Thermal deflections/stresses.

#### 3.1 The thermal bowing deflection at mid-height of a steel stud fire wall

A firewall comprises a fire protecting layer of plasterboard either side of an assembly of steel studs. The vertical studs are of a light steel channel section 200mm deep between flanges and the studs are 6m high and simply supported at their ends. In an indicative fire resistance test on a 1m high specimen of the whole construction exposed to the ISO 834 fire on one face it was found that the thermocouple temperatures of the ‘hot’ and ‘cold’
flanges of the steel channel were 700 and 300°C respectively. The two conditions considered are shown in the figure below.

Assuming a) the plasterboard does not affect the bowing behaviour, and b) the steel properties are unaffected, what is the mid-height deflection \( \Delta_{\text{mid\text{-}height}} \) of the 6m high wall in the horizontal direction assuming the same fire exposure?

We see that we can assume the ends of the studs are free to rotate and that the temperature difference across the stud channel section is the same as in the test.


\[
\Delta_{\text{mid\text{-}height}} = \alpha \frac{H^2(T_1 - T_2)}{8d}
\]

Where

- \( \alpha \) = coefficient of linear expansion \( (14 \times 10^{-6} / ^\circ\text{C} \) for steel at elevated temperature)  
- \( H \) = height of member (m)  
- \( T_1 \) = temperature of ‘hot’ face \( (^\circ\text{C}) \)  
- \( T_2 \) = temperature of ‘cold’ face \( (^\circ\text{C}) \)  
- \( d \) = distance between faces (m)
Substituting values in the equation gives:

\[ \Delta_{\text{mid-height}} = \frac{14 \times 10^{-6} \times 6^2 \times (700 - 300)}{8 \times 0.20} = 0.126 \text{m} \]

### 3.2 The thermal bowing deflection at the top of a steel stud firewall

Using the same data as for the problem above, calculate the horizontal deflection at the top of the wall (\(\Delta_{\text{top}}\)) assuming it is a cantilever (i.e. it is direction-fixed (encastre) at the base and free at the top to move both laterally and vertically)

From the literature (as above example)

\[ \Delta_{\text{top}} = 4 \times 0.126 = 0.504 \text{m} \]

### 3.3 The thermal bowing deflection at the top of a tall steel stud firewall in a simulated fire resistance test

In a fire resistance test the measured horizontal deflection of a 3m high partition at mid-height was 200mm. What would be the corresponding deflection for a partition of the same construction 6m high, assuming the same fire exposure conditions and only geometrical thermal bowing effects?
From the governing equation (as above example):

\[ \Delta_{\text{mid-height}} = \frac{\alpha H^2 (T_1 - T_2)}{2d} \]

, it is clear that deflection varies as the square of the height.

Therefore:

\[ \Delta_{\text{mid-height}} = 200 \left( \frac{6^2}{3^2} \right) = 800\text{mm} \]

3.4 The lateral bowing of a heated slender steel member

A very slender vertical steel rod 2m long is position-fixed at both ends, i.e. the rod is pin-ended and the ends cannot move axially. It is then heated uniformly along the whole length through a temperature of 400°C. Determine the lateral bowing deflection at mid-length assuming a) the rod bows into a circular arc and b) the rod does not shorten due to compression caused by axial restraint at the ends.
3.5 The hypothetical expansion-restraint force for a squat steel section

A solid steel section is 150mm square and 0.5m long. It is longitudinally restrained at its ends so that it is unable to expand longitudinally. It is heated from room temperature (say 20°C) to 700°C, see figure below. What expansion force is developed, assuming elastic behaviour and that buckling does not occur?

Elastic Modulus $E = \frac{\text{stress}}{\text{strain}} = \frac{P}{A} \cdot \frac{\Delta}{L}$ \hspace{1cm} (1)

Where $P =$ axial force (N)

$A = $ Cross-section area (mm$^2$)

$\Delta =$ unrestrained expansion (mm)

$L =$ length of member (mm)

\[ D = L(0.375\alpha T)^{1/2} \]

\[ D = 2(0.375 \times 14 \times 10^{-6} \times 400)^{1/2} = 0.0916 \text{m} \]
The unrestrained expansion is given by:

\[ \Delta = \alpha L (T_1 - T_2) \]  

Where \( \alpha \) = coefficient of linear thermal expansion (14x10^-6/oC for steel)  
\( T_1 \) = temperature when hot (°C)  
\( T_2 \) = temperature when cold (°C)

Substituting (2) in (1) and rearranging gives:

\[ P = \alpha AE (T_1 - T_2) \]  

Assume \( E_{700} = 0.3 \) of the room temperature value and assume room temperature value is 200kN/mm²

From Equation (3)

\[ P = 14 \times 10^{-6} \times 150^2 \times 0.3 \times 200,000 \times 680 = 12,852 \text{kN} \]

Note that this is a massive force and in practice the end restraints would probably move apart and elastic and plastic deformation in the steel section would occur.

4 Fire Fighting calculations

4.1 The pressure at the highest outlet of a wet riser
A wet riser is provided in an 8-storey building with an outlet at every storey. The highest outlet is 26m above the fire fighting pump, see Figure below. Assuming the available pressure from a fire fighting pump is 8 bar, what is the water pressure at the highest riser outlet assuming it is in use and is required to provide a flow of 22 l/s? Ignore losses in the hose connecting the pump to the riser inlet.

**Figure. Wet riser and pump diagram**

Data: the developed length of the steel pipe work from pump outlet to the highest outlet from the wet riser is 65m and the pipe is 100 mm internal diameter. The loss of head due to friction is assumed to be 0.1m/m at a flow of 22 l/s. Note that 1 bar pressure (100,000N/m² or 14.5 pounds/square inch) corresponds approximately to 10m head of water.

Available static head at highest outlet (i.e. no flow) = 8 – 26/10 = 5.4 bar

Friction loss in pipe = length of pipe x friction loss per metre = 65 x 0.1 = 6.5m which corresponds to 0.65 bar

Therefore available pressure at highest outlet with flow of 22 l/s = 5.4 – 0.65 = 4.75 bar

**4.2 The pressure needed at a pump appliance to get a good jet of water.**

A fireman’s jet with a nozzle of 20mm internal diameter is being used on the 6th floor of a building in which there is no fire main. Five lengths of 70mm diameter hose each 25m long are connected and run from the pump appliance up the staircase to the fire floor (the 5th floor which is 15m above the pump appliance). The pressure required at the nozzle to get a good jet of water is assumed to be 4 bar. What pressure is required at the pump appliance outlet?
To raise the water through 15m requires a pressure of 1.5 bar. The pressure loss in 5 lengths of hose assuming a loss of 0.2 bar/length = 1.0 bar. Therefore pressure needed at pump = 4.0 + 1.5 + 1.0 = 6.5 bar.

4.3 The effect of changing the hose nozzle diameter on the range of a fire fighting jet

A fireman’s hose with a nozzle outlet of 20mm internal diameter has a range, say, of 25m for a pressure of 3.5bar at the nozzle (range is the horizontal distance to which most of the water is thrown when the nozzle is at the optimum angle, usually around 35° to the horizontal). If the nozzle is changed for one with a 25mm internal diameter nozzle, what will be the range assuming the same pressure at the nozzle?

![Fireman's hose water range](image)

From the literature (BRE IP 16/84) the relationship between range R, pressure p, and diameter d, is given by:

\[ R = 5.47p^{0.42}d^{0.46} \]

Hence we can say that \( \frac{R_1}{R_2} \) is proportional to \( \frac{d_1}{d_2} \) so that \( R_1 = R_2 \left( \frac{d_1}{d_2} \right)^{0.46} \)

Substituting values gives: \( R_1 = 25 \left( \frac{20}{25} \right)^{0.46} = 22.5 \text{m} \)

4.4 Sucking water up from a low level reservoir for fire fighting

A pump appliance has access to water in a reservoir which is suitable for fire fighting purposes. The level of the water is 11m below the pump appliance and sufficient length of hard hose is available to reach it. What suction pressure is needed to lift the water up to the appliance?
The question is meaningless since it is impossible in practice to lift water up through a pipe by more than 8m without getting cavitation.

5 Thermal radiation

5.1 Simplification of configuration factors for thermal radiation calculations

Introduction.

It is sometimes necessary in fire safety engineering projects to make calculations of radiation intensity. This may be necessary, for example, when buildings are close to each other and there is a possibility that fire can spread by radiation from one building to the other. Or, within a building, there may be fuel packages near each other which need to be separated, but by how much?

The calculation requires the use of radiation configuration factor as will be shown below. The configuration factor can be numerically tedious to calculate and it is advantageous to use simple equations where possible. The following note determines the errors in configuration factors which have been simplified. The stimulus for this work came from engaging in the work of ISO committee SC4 of TC 92 ‘Fire Safety Engineering’ and the note is the author’s contribution to the work in mid-2008.

I asked myself the question ‘Might it be possible to use an equation for the configuration factor which is less complex than that relating to a cylinder (the cylinder represents an idealization of a flame in this case) while retaining reasonable accuracy?’ For example could the equation for a rectangle or, simpler still, the equation for an ellipse be used? This note indicates that this is feasible.

Objective and assumptions.

This paper examines the numerical error in the value of a radiation configuration factor if it is assumed that a cylinder can be considered as a rectangle or an ellipse of the same overall geometric size and same separation distance between radiator and receiver (target). In all cases:

- the receiver is parallel to the radiator
- the receiver is opposite the centreline of the radiator at the bottom
- the radiator is isothermal.

Background

The radiation intensity \( I_1 \) emitted from a hot body is related to the absolute temperature \( T \) of the radiator by the following equation:

\[
I_1 = \varepsilon \sigma T^4
\]  

(1)

where:
\[ \varepsilon = \text{emissivity of the surface of the radiator (value from 0 to 1)} \]
\[ \sigma = \text{Stefan-Boltzmann constant (5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4})} \]

The intensity of radiation received \((I_2)\) at a point some distance from the radiator is geometrically related to the emitted radiation intensity by a radiation configuration factor, sometimes called the view factor, \((\phi)\) such that:

\[ I_2 = \phi I_1 \]  

(2)

The smaller the value of \(\phi\) the smaller the intensity of radiation received. \(\phi\) has a value between 0 and 1 and can be visualized as the solid angle seen when looking at the radiator from the receiver (target).

Configuration factors given in text books display varying amounts of complexity and sometimes include errors in their reproduction over the years. The practicing fire engineer would prefer to use a simple and reliable equation where there is a choice. As will be seen below, the configuration factor for a cylindrical shape is relatively complex and it is easy to introduce errors when substituting numerical values. Some simplification and small inaccuracy is considered acceptable when considering the other simplifying assumptions used in fire engineering – assumptions, for example, on size, shape and orientation of flame, flame emissivity and temperature.

**Nature of comparative analysis**

In the following analysis the radiation is assumed to be produced by a pool fire of cylindrical form of radius \(r\). The receiver (target) ‘sees’ only a portion of the cylindrical fire (portion represented by line D when seen from point S in Figure 1). If, for simplicity in calculating the configuration factor, the fire is assumed to be represented by a flat rectangular surface, the question then arises ‘Where should the flat radiator be positioned relative to the receiver to get roughly the same value of configuration factor as for the cylinder?’

Figure 1 shows a plan view of the cylinder and 3 positions of a flat radiator of width \(2r\) at positions A, B and C. It is noted that when the receiver is close to the radiator, i.e. at point S, only a portion of the surface of the cylinder is ‘seen’ by the receiver i.e. a width less than \(2r\) and this is indicated by arc line D in Figure 1; note also that the separation distance varies along the arc. The separation distances for the three positions of the
rectangular flat radiator are shown in the figure in relation to a close receiver at \( S \).

![Figure 1. Plan view showing the radiator conditions considered (rectangles at stations A, B, C and cylinder at arc line D) ![Figure 1. Plan view showing the radiator conditions considered (rectangles at stations A, B, C and cylinder at arc line D)](image)

It is clear that a flat radiator at \( C \) will lead to an over estimate of configuration factor and at \( A \) will lead to an under estimate when compared to the cylindrical surface. By inspection of Figure 1, for a close receiver it seems position \( B \) would be a good choice. It should be noted that Figure 1 shows the receiver at \( S \), close to the radiator, and this will lead to differences between the four conditions; with large separation distances (with the receiver at point \( T \)) the differences will be less noticeable.

It has also been suggested (Tanaka) that the cylindrical radiator and the rectangular radiator can both be considered as an ellipse having the same major and minor axis dimensions as the rectangle, and this has the advantage that the equation for the configuration factor is further simplified. The effect of using the ellipse has also been examined in the comparisons below.

**The cylinder as radiator**

The equation for the configuration factor \( \phi \) for a cylinder is given by Hamilton and Morgan [1] by:

\[
\phi = \frac{1}{\pi D} \tan^{-1} \left( \frac{L}{\sqrt{D^2 - 1}} \right) + \frac{L}{\pi} \left[ \frac{A - 2D}{D\sqrt{AB}} \tan^{-1} \left( \frac{A(D - 1)}{B(D + 1)} \right) - \frac{1}{D} \tan^{-1} \left( \frac{D - 1}{D + 1} \right) \right] \tag{3}
\]

Where \( D = \frac{d}{r} \), \( L = \frac{l}{r} \), \( A = (D + 1)^2 + L^2 \), \( B = (D - 1)^2 + L^2 \).

For the practising fire safety engineer who wishes to make a one-off calculation the calculation using Equation (3) is prone to error even if the correct equation is used. Also the equation found in the literature may be incorrect due to typographical errors.
For example, the equation given in Howell’s catalogue [2] is different to the equation cited by Hamilton and Morgan though similar in form. The Hamilton and Morgan equation has been adopted in the present paper as it stems from an authoritative and important reference and was tabulated for a range of values of D and L.

Furthermore the Hamilton and Morgan equation is adopted in the SFPE Handbook of Fire protection Engineering, 3rd edition though, strangely, with different nomenclature (In the SFPE handbook, L is said to equal L/R which is an impossibility, and there is no L and R indicated in the accompanying SFPE diagram). [Inform editor of SFPE handbook]

**The rectangle as radiator**

For the area bounded by ABCD, Figure 3, the equation for the configuration factor \( \phi \) is given [2] by:

\[
\phi = \frac{1}{2\pi} \left\{ \frac{x}{\sqrt{x^2 + y^2}} \tan^{-1} \frac{z}{\sqrt{x^2 + y^2}} + \frac{z}{\sqrt{z^2 + y^2}} \tan^{-1} \frac{x}{\sqrt{z^2 + y^2}} \right\}
\]  

(4)
By inspection it is clear that this is a relatively simple equation to solve as there are only three parameters to consider and these may be easily entered into a spread sheet.

Since configuration factors are additive and subtractive we can see from Figure 3 that the configuration factor for area bounded by ABEF is twice that for area ABCD.

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**The ellipse as radiator.**

Tanaka has suggested [4] that for many practical fire engineering calculations the configuration factor for an ellipse can be used in place of the configuration factor for a rectangle. The original source of the configuration factor equation for the ellipse is not known and the equation given below is therefore that given in Tanaka’s paper. For the whole area of the ellipse shown in Figure 4, in which the radiator and target are parallel, as in all the comparisons, the equation for the configuration factor $\phi$ is given by Equation (5):

$$\phi = \frac{ab}{\sqrt{(s^2 + a^2)(s^2 + b^2)}}$$  \hspace{1cm} (5)
Figure 4 Configuration parameters for an elliptical radiator with a parallel receiver at A or B.

It should be noted that Equation (5) assumes the target is at A opposite the centre of the ellipse, not at the base B, as in the other comparisons made. To correct for this, the value of $\phi$ has been halved.

**Comparison of configuration factor for rectangular and cylindrical radiators (rectangle coincident with diameter).**

To facilitate the direct comparison of configuration factor for cylinder and rectangle, the width of the rectangle, $2x$, should equal the diameter of the cylinder, $2r$, and the height of the rectangle, $z$, should equal $d$, using the nomenclature in Figures 2 and 3.

The comparisons have been made for $r = x = 1$ and $z = l = 4$. Therefore the rectangle is 2 units wide by 4 units high and the cylinder is also 2 units diameter and 4 units high.

In this case the rectangle is positioned at the cylinder diameter i.e. at station A in Figure 1, and the comparison, Figure 5, shows that the configuration factor for the cylinder is greater than that for the rectangle at small separation distances. This is intuitively as expected because the front face (nose) of the cylinder is closer to the receiver than the face of the rectangle (in the latter all the radiating surface is in the same plane and coincident with the diameter of the cylinder). Hence the use of the configuration factor for a rectangular radiator coincident with the diameter would not be conservative i.e. it would be unsafe.
Comparison of configuration factor for rectangular and cylindrical radiators (rectangle nearer to target by radius/2)

Again, from Figure 1, it seems intuitive that placing the rectangle at station B, i.e. at r/2, would provide a better equivalent position of the rectangular radiator. The comparison is shown in Figure 6 and indicates very good agreement.

Comparison of configuration factor for elliptical, rectangular and cylindrical radiators (rectangle and ellipse at station A, Figure 1)

Here again the ellipse and rectangle have the same overall size as the cylinder. The ellipse and rectangle are located at station A, Figure 1, i.e. on the diameter. The comparison, in Figure 7, shows reasonably good agreement, but the ellipse, like the rectangle, slightly underestimates the value of configuration factor, but this may be acceptable bearing in mind the other uncertainties in radiation calculations.
Conclusions.

Fire engineers have to make many simplifications in their work. For example, in radiation calculations it is helpful to be able to use a simple equation for a configuration factor to reduce the possibility of numerical error, especially important in one-off fire safety engineering calculations. It is clear that the equation for the configuration factor of an ellipse is simple whereas that for a cylinder is relatively complex.

It has been shown, Figure 6, that a cylindrical radiator (representing, for example, a circular pool fire), can be considered to be a rectangular radiator in the calculation of radiation configuration factor with little error if the separation distance is taken as the distance from the cylinder centerline to the receiver minus half the radius of the cylinder.

It also appears, Figure 7, that a cylindrical or rectangular radiator can be considered to be an elliptical radiator and this further simplifies the calculation (compare the complexity of Equations 3, 4 and 5).

The present author has also calculated the value of configuration factor for a rectangular radiator and a cylinder of same size using a large separation distance where it is clear that the two equations (Equations 2 and 3) should give approximately the same value. Using a common separation distance of 20 and adopting the values used in the above comparison (ie a height of 4 and a width of 2) gave configuration factors of 0.00644 and 0.006191 for the cylinder and the rectangular radiator respectively, a trivial difference of less than 4%.

Note of caution.

This analysis does not consider height and width of radiator as variables. Only the separation distance has been considered as a variable.

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References


